TAILURING PROCEDURE

Progress Report

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Aditi Chattopadhyay Associate Professor Department of Mechanical and Aerospace Engineering Arizona State University Tempe, Arizona 85287-6106

Objectives

Composite wings have become increasingly popular in recent years due to significant potential of weight savings and stiffness tailoring. Therefore, a composite box beam is being used to model the principal load carrying member of an airplane wing. A new composite beam modeling technique is being developed to ensure accuracy. The procedure will be used to investigate the effect of composite tailoring on aeroelastic stability of airplane wing using formal design optimization techniques.

Approach

Several approaches addressing composite box beam modeling have been proposed over the last few years. All of these models have several limitations. In some of these work, classical laminate theory (CLT) was used and transverse deformations through the wall thickness were neglected to make the contour analysis easy. For composite structures in which strong elastic coupling exists, these transverse stresses and strains heavily influence the structural behavior. Also, in aircraft applications, the assumption of thin-walled sections is not relevant when composite stiffeners are used. Therefore, it is necessary to develop analysis techniques which can account for cross sections with arbitrary wall thicknesses. Secondly, in some of the work, the cross section geometry was assumed to remain rigid during beam deformation and thus in-plane warping was neglected. However, in-plane warping is important for loaded wing structures with short aspect ratio. In the existing literature, the out-of-plane warping is also assumed to be small and with linear variations which is superimposed upon large rigid cross-sectional deformations without proper justifications. The above drawbacks make these models inadequate for mere general applications. Therefor, in the present research, a higher order composite laminate theory is being used to model the displacement field including through-the-thickness variation of transverse deformations. Both in- and out-of-plane warping are included in the model. The procedure is briefly described below.

Refined displacement field: The individual walls of the box beam are modeled as plates with arbitrary thicknesses. Using the higher order theory, the through-the-thickness displacement distributions of the plate (Fig. 1) are expressed as follows:

$$u = u_0(x, y) + z \, \psi_X(x, y) + z^2 \, \zeta_X(x, y) + z^3 \, \phi_X(x, y)$$

$$v = v_0(x, y) + z \, \psi_Y(x, y) + z^2 \, \zeta_Y(x, y) + z^3 \, \phi_Y(x, y)$$

$$w = w_0(x, y) + z \, \psi_Z(x, y) + z^2 \, \zeta_Z(x, y) + z^3 \, \phi_Z(x, y)$$
(1)

where u, v and w represent the displacements in x, y and z directions respectively (Fig. 1). On the right side of Eqn. (1), u0, v0, and w0 denote the displacements of a point (x,y) on the reference plane, and ψ_x , ψ_y and ψ_z are the rotations of normals with respect to reference plane. The functions ζ_x , ζ_y , ζ_z , ϕ_x , ϕ_y and ϕ_z represent higher order terms which are neglected in classical laminate theory (CLT). The transformation of the higher order plate displacement field into beam displacement is described below.

The displacement field of every wall is described using the form given by Eqn. (1). Therefore, using simple superposition, the displacement of an arbitrary point on the box beam cross section (Fig. 2) is written as follows.

$$u = u_{10}(x, y, 0) + z \psi_{1x}(x, y) + z^{2} \zeta_{1x}(x, y) + z^{3} \phi_{1x}(x, y) + u_{20}(x, 0, z) + y \psi_{2x}(x, z) + y^{2} \psi_{2x}(x, z) + y^{3} \phi_{2x}(x, z)$$

$$v = v_{10}(x, y, 0) + z \psi_{1y}(x, y) + z^{2} \zeta_{1y}(x, y) + z^{3} \phi_{1y}(x, y) + v_{20}(x, 0, z) + y \psi_{2y}(x, z) + y^{2} \psi_{2y}(x, z) + y^{3} \phi_{2y}(x, z)$$

$$w = w_{10}(x, y, 0) + z \psi_{1z}(x, y) + z^{2} \zeta_{1z}(x, y) + z^{3} \phi_{1z}(x, y) + w_{20}(x, 0, z) + y \psi_{2z}(x, z) + y^{2} \psi_{2z}(x, z) + y^{3} \phi_{2z}(x, z)$$
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Taylor series expansions are then used to reduce redundancy in the above expression. After using an ordering scheme and collecting terms, the beam displacement field is now expressed as follows.

$$u(x, y, z) = u_0(x) + z a_X(x, y) + z^2 b_X(x, y) + z^3 c_X(x, y) + y d_X(x, z) + y^2 e_X(x, z) + y^3 f_X(x, z)$$

$$v(x, y, z) = v_0(x) + z a_Y(x, y) + z^2 b_Y(x, y) + z^3 c_Y(x, y) + y d_Y(x, z) + y^2 e_Y(x, z) + y^3 f_Y(x, z)$$
(3)

$$w(x, y, z) = w_0(x) + z a_z(x, y) + z^2 b_z(x, y) + z^3 c_z(x, y) + y d_z(x, z) + y^2 e_z(x, z) + y^3 f_z(x, z)$$

where u(x, y, z), v(x, y, z) and w(x, y, z) represent the beam displacement functions along x, y, and z directions, respectively. The quantities $u_0(x)$, $v_0(x)$ and $w_0(x)$ are the displacements of the original point in the coordinate system described in Fig. 2. The functions $a_i(x, y)$, $b_i(x, y)$, $c_i(x, y)$, $d_i(x, z)$, $e_i(x, z)$ and $f_i(x, z)$ where i = x, y, z, represent combinations of rigid cross-sectional rotations and both in- and out-of-plane warpings.

The final step is to make y and z explicit and displacement functions only implicit along x. Noticing that the sine and cosine functions are basic solutions (mode shapes) for the plate bending problem, all the y dependent functions are expanded along beam width and all the z dependent functions are expanded along beam height. This reduces the complexity of the problem since the y and z dependencies are now expressed explicitly. The displacement functions are now only implicit along x. Expansions are based on first five basic mode shapes of the box beam cross section. Thus, a quasi one-dimensional beam displacement field with 93 x dependent unknown functions is obtained.

Numerical solution: Structural response under prescribed load are calculated using the finite elements method. Within each element, the quasi one-dimensional displacement field is interpolated using spanwise cubic shape functions which results in 372 unknowns per element. A weak formulation based on Lagrangian approach is used to arrive at the following dynamic equations of motion:

$$M\ddot{q} + C\dot{q} + Kq = F \tag{4}$$

where M, C and K are the mass, the damping and the stiffness matrices, respectively and F is the forcing function corresponding to the nodal degrees of freedom q.

Results

At present results have been obtained for both first and second order theories by retaining second order terms in the displacement fields (Eqn.1). Computations have been performed on composite beams with different wall thickness and slenderness (Fig. 1 and Table 1) using both the first and the second order model. Two stacking sequence configurations of (0/90°) symmetric and (45°/-45°) symmetric have been used for the walls of the composite beam. The later configuration results in the bending/twist coupling. A vertical load is applied at every beam tip.

Results from the new theories are compared with those obtained using a model based on the classical laminate theory (CLT) and are presented in Tables 2 and 3. This simplified model ignores extension and shear terms and only considers bending about the y axis and torsion about the x axis. Thus it under predicts beam bending and twist since the transverse in plane strains in the horizontal and vertical laminates are not accounted for in the analysis. From Tables 2 and 3 it is observed that both the first and the second order models show significant improvements over the simplified CLT model, particularly for short beams where stronger bending/twist coupling exists. For slender beams shear strain has little influence on bending deformation and therefore better agreement between the theories exists.

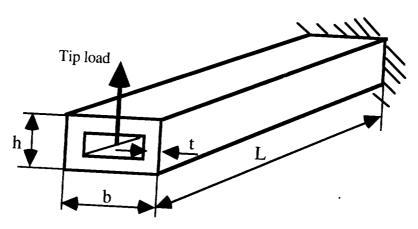


Figure 1 Box beam with tip load

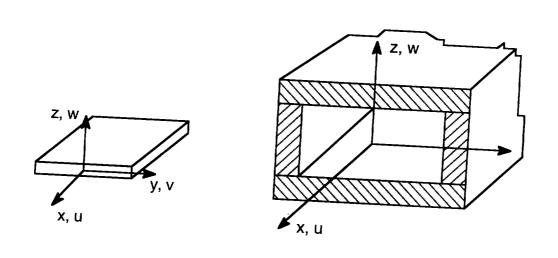


Figure 2. Box beam cross section

Table 1 Beam dimensions

Beam No.	L (mm)	b (mm)	h (mm)	t (mm)	Ply stacking
1	200	50	20	0.6	[0/90°/0] _s
2	500	50	20	0.6	[45°/-45°/45°]s
3	80	20	20	2	[0/90°/0/90°/0] _S
4	80	20	20	2	[45°/-45°/45°/-45°/45°]s

Table 2 Comparison of results

Beam No.	Tip Displacement (mm)				
	CLT	1st. Order	2nd. Order		
		Theory	Theory		
1†	0.03595	0.03976	0.04383		
		(10.6%)*	(21.9%)*		
2†	1.18	1.34	1.49		
		(13.6%)*	$(26.3\%)^*$		
3 †	0.001548	0.003821	0.004812		
		(146.8%)*	(210.9%)*		
4††	0.4531	0.5771	0.7625		
		$(27.4\%)^*$	(68.3%)*		

Table 3 Comparison of results

Beam No.	Tip Twist (degree)				
	CLT	1st. Order Theory	2nd. Order Theory		
2†	-0.00015	-0.00036	-0.0004		
4††	-0.00015	(140.0%)* -0.00087	(166.7%)* -0.0011		
		(433.3%)*	(633.3%)*		

[†] Deformation under unit tip load. †† Deformation under 100N tip load. * Percentage deviation from CLT.